



CEMC.UWATERLOO.CA | The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

Grade 7/8 Math Circles

November 28/29/30/December 1, 2022

Jeopardy

Recursive Sequences - 100

Find an infinite sequence which is both an arithmetic sequence and a geometric sequence.

Answer

Recursive Sequences - 200

Find the next 2 numbers in the following pattern:

1, 21, 312, 4213, ...

Answer

Recursive Sequences - 300

The sequence t_1, t_2, t_3, \dots is geometric.

If $t_1 = 3$ and $t_4 = -24$, find t_{10} .

Answer

Recursive Sequences - 400

Let a_1, a_2, a_3, \dots be an arithmetic sequence with first term 7 and common difference 13. Find n such that $a_n = 2022$.

Answer

Recursive Sequences - 500

Let a_n be the number of ways one can climb an n -step staircase by going up either one or two steps at a time. Fill out the following table.

n	1	2	3	4	5	6	7	8
a_n	1	2						

Answer

Recursive Sequences - 100

Answer

The sequence of 0's: 0, 0, 0, 0, ...

Recursive Sequences - 200

Answer

1, 21, 312, 4213, 53124, 642135, ...

Recursive Sequences - 300

Answer

$$t_{10} = -1536$$

Recursive Sequences - 400

Answer

$$n = 156$$

Recursive Sequences - 500

Answer

n	1	2	3	4	5	6	7	8
a_n	1	2	3	5	8	13	21	34

Math Logic - 100

- I will eat either pizza or spaghetti for dinner.
- I will not eat spaghetti for dinner.

What would I have for dinner?

Answer

Math Logic - 200

What is the mathematical symbols for “or”, “and”, ”not”, in that order?

Answer

Math Logic - 300

Make a truth table for $P \vee Q$.

Answer

Math Logic - 400

Find the simplest equivalent logical expression of $\neg(\neg\neg P \vee \neg Q) \wedge Q$.

Answer

Math Logic - 500

Four boys, Joshua, Daniel, Nicholas, and Ryan, are at home to watch some movies. Who does like Action movies?

1. Joshua is at one of the ends.
2. The boy wearing the Black shirt is somewhere to the left of the youngest boy (11-year-old).
3. Joshua likes Horror movies.
4. The 14-year-old boy is at the third position.
5. The boy wearing the Red shirt is somewhere between the 13-year-old boy and the one who likes Action movies, in that order.
6. Daniel likes Thriller movies.
7. The boy who is going to eat Cookies is at one of the ends.
8. The boy wearing the Black shirt is exactly to the left of the one who likes Thriller movies.
9. The boy who is going to eat Crackers is exactly to the right of the boy who likes Comedy movies.
10. The boy wearing the Red shirt is somewhere between the boy who is going to eat Popcorn and Nicholas, in that order.
11. At one of the ends is the boy who likes Thriller movies.
12. Nicholas is somewhere between Joshua and Daniel, in that order.
13. At the first position is the boy wearing the Green shirt.

Answer

(Taken from <https://www.brainzilla.com/>)

Math Logic - 100

Answer

Pizza!

Math Logic - 200

Answer

\vee, \wedge, \neg

Math Logic - 300

Answer

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Math Logic - 400

Answer

$$\neg P \wedge Q$$

Math Logic - 500
Answer

Nicholas

Radians - 100

Let r be the radius, let d be the diameter, let C be the circumference, and let A be the area.

- a) What is the formula for the circumference of a circle?
- b) What is the formula for the area of a circle?

Answer

Radians - 200

Why was 360° chosen to be the number of degrees in a full circle? List at least two theories.

Answer

Radians - 300

Convert 20° to radians.

Answer

Radians - 400

Calculate the arc length of the sector with radius 28 cm and a central angle of $\frac{27\pi}{14}$ radians.

Answer

Radians - 500

Calculate the sector area of the sector with radius 11 mm and a central angle of $\frac{4\pi}{3}$ radians.

Answer

Radians - 100

Answer

a) $C = 2\pi r$ or $C = \pi d$

b) $A = \pi r^2$

Radians - 200

Answer

- Highly composite
- Solar and lunar years
- Sexagesimal (base 60) number system and equilateral triangles

Radians - 300

Answer

$$20^\circ = \frac{\pi}{9} \text{ radians}$$

Radians - 400

Answer

$$\text{arc length} = 54\pi \text{ cm}$$

Radians - 500

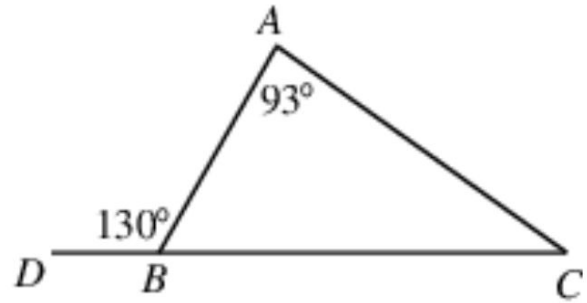
Answer

$$\text{sector area} = \frac{242\pi}{3} \text{ mm}^2$$

Gauss Prep - 100

In the diagram, the size of $\angle ACB$ is

- (A) 57° (B) 37° (C) 47°
(D) 60° (E) 17°



(Source: 2005 Gauss (Grade 7), #8)

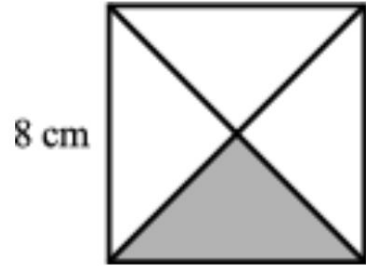
Answer

Gauss Prep - 200

The diagonals have been drawn in the square shown. The area of the shaded region of the square is

- (A) 4 cm^2 (B) 8 cm^2 (C) 16 cm^2
(D) 56 cm^2 (E) 64 cm^2

(Source: 2014 Gauss (Grade 7), #12)



Answer

Gauss Prep - 300

How many positive whole numbers, including 1, divide exactly into both 40 and 72?

(A) 9

(B) 12

(C) 4

(D) 2

(E) 5

(Source: 2007 Gauss (Grade 7), #15)

Answer

Gauss Prep - 400

There are several groups of six integers whose product is 1. Which of the following cannot be the sum of such a group of six integers?

- (A) -6 (B) -2 (C) 0 (D) 2 (E) 6

(Source: 2018 Gauss (Grade 7), #18)

Answer

Gauss Prep - 500

In the addition shown, P and Q each represent single digits, and the sum is $1PP7$.
What is $P + Q$?

$$\begin{array}{r} 77P \\ 6QP \\ + \quad QQP \\ \hline 1PP7 \end{array}$$

(A) 9

(B) 12

(C) 14

(D) 15

(E) 13

(Source: 2013 Gauss (Grade 7), #21)

Answer

Gauss Prep - 100
Answer

(B)

Gauss Prep - 200
Answer

(C)

Gauss Prep - 300

Answer

(C)

Gauss Prep - 400

Answer

(C)

Gauss Prep - 500
Answer

(C)

Induction - 100

Let t_1, t_2, \dots be a sequence such that $t_{n+1} = t_n + 1$ for all natural numbers n . Find $t_8 - t_1$.

Answer

Induction - 200

Find all $n \in \mathbb{N}$ such that $2^n \leq n^2$.

Answer

Induction - 300

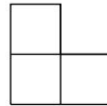
Compute the sum of the first 50 odd positive integers:

$$1 + 3 + 5 + \dots + 97 + 99$$

Answer

Induction - 400

Find all natural numbers n such that a $3 \times n$ grid of squares can be tiled using triominoes.
Recall: a triomino is an L-shaped tile (see below).



Answer

Induction - 500

Let a_1, a_2, a_3, \dots be a sequence of positive real numbers defined by $a_1 = 1$ and $a_{n+1} = \sqrt{a_n + 2}$ for $n \geq 1$. True or False:

- a) $a_2 \leq 2$
- b) For all positive real numbers $x \leq 2$, $\sqrt{x + 2} \leq 2$.
- c) $a_n \leq 2$ for all $n \in \mathbb{N}$.

Answer

Induction - 100

Answer

$$t_8 - t_1 = 7$$

Induction - 200

Answer

$$n = 2, 3, 4$$

Induction - 300

Answer

$$50^2 = 2500$$

Induction - 400

Answer

All even natural numbers n .

Induction - 500

Answer

a) True.

b) True.

c) True.

Modular Arithmetic - 100

What is the remainder of $-347 \div 5$?

Answer

Modular Arithmetic - 200

Is 11 congruent to 87 modulo 3?

Answer

Modular Arithmetic - 300

What is the last digit of $3829^2 \times (2891^{92} - 47728 + 2692213^3)$?

Answer

Modular Arithmetic - 400

What is the remainder when $(189^5 + 3417^{283}) \times 164^2$ is divided by 17?

Answer

Modular Arithmetic - 500

- a) If today is Tuesday, what day is it in 589 days?
- b) If today is Friday, what day was it 264 days ago?

Answer

Modular Arithmetic - 100

Answer

3

Modular Arithmetic - 200
Answer

No

Modular Arithmetic - 300
Answer

0

Modular Arithmetic - 400
Answer

13

Modular Arithmetic - 500

Answer

- a) Wednesday
- b) Sunday

Complex Numbers - 100

What is the imaginary part of the number 0?

Answer

Complex Numbers - 200

Calculate/simplify i^{77} .

Answer

Complex Numbers - 300

Calculate $(-5 + 7i) \times (2 - 3i)$

Answer

Complex Numbers - 400

Calculate $\frac{-5 + 7i}{2 - 3i}$

Answer

Complex Numbers - 500

Find solutions of the quadratic equation $2x^2 + 4x + 7 = 0$.

Answer

Complex Numbers - 100
Answer

0

Complex Numbers - 200

Answer

i

Complex Numbers - 300

Answer

$$11 + 29i$$

Complex Numbers - 400

Answer

$$\frac{-31 - i}{13}$$

Complex Numbers - 500

Answer

$$-1 \pm \frac{\sqrt{10}}{2}i$$